STABILITY OF BOUNDARY MOTION FOR AN EXPLOSION-PRODUCED CAVITY IN SOFT GROUND

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One of the first models of an explosion in the ground was a hydrodynamic scheme [1]. Despite the deficiencies and definite thermodynamic contradictions [2] of the scheme, the qualitative picture of an explosion in the ground [1] is described rather clearly. Subsequently, many papers appeared which were concerned with this problem and which used more complex plastic and elastoplastic models as simulated media, but the model [1] of an explosion in the ground, as before, impresses one with its simplicity. Using the formulation of [1], the present paper investigates the stability of the boundary of an explosion-produced cavity and of the shock wave. Consideration is limited to the simplest case where the main motion is one-dimensional.

In the present case, the scheme of [1] takes the following form. A shock wave starts propagating at the time t = 0 in a medium with a constant density ρ_1. Behind the shock front there is an incompressible ideal fluid with a density ρ_2. The solutions characterizing the motion of the medium are of the form

$$\frac{\partial u_0}{\partial t}+u_0\frac{\partial u_0}{\partial x}=-\frac{1}{\rho_2}\frac{\partial p_0}{\partial x};\quad \frac{\partial u_0}{\partial x}=0,$$

where u_0 is the projection of the velocity vector on the x axis and p_0 is the pressure. At the boundary $x = a$ of the explosion-produced cavity, the pressure in the fluid is equal to the pressure in the cavity. At the shock front x = R, the conditions

$$u_0(R)=\xi\dot{R};$$

$$p_0(R)=\rho_1\xi\dot{R}^2+p_s,$$

are satisfied, where $\xi = 1-\rho_1/\rho_2$, the dot denotes differentiation with respect to time, and p_s is the pressure from which shock compression is initiated. Unperturbed flow will be characterized by the following relations:

$$a=\xi R+(1-\xi)a_0,\quad a_0=R_0 \text{ — is the initial state};$$

$$p_0=-\rho_2\dot{u}_0x+\frac{\rho_1u_0^2}{\xi}+\rho_2\dot{u}_0R.$$

We assume that the main flow is subjected to a small perturbation

$$U_x=u_0+u;\ U_y=v;\ p=p_0+p;^*$$

$$X_b=R+\varepsilon;\ X_n=a+\delta,$$

where U_x and U_y are the components of the velocity vector of the perturbed flow; $X_n(y, t)$ and $X_b(y, t)$ are, respectively, the equations for the boundaries of the explosion-produced cavity and the shock wave.

*As in Russian original. – Publisher.

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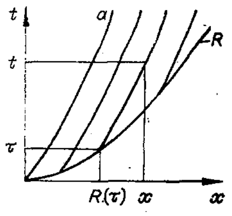

Fig. 1

The equations of motion and the equation of continuity, linearized with respect to the perturbation, are written in the form

$$\begin{cases} \frac{\partial u}{\partial t} + u_0 \frac{\partial u}{\partial x} = -\frac{1}{\rho_2} \frac{\partial p}{\partial x}; \\ \frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} = -\frac{1}{\rho_2} \frac{\partial p}{\partial y}; \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \end{cases} \quad (1)$$

Linearization of the conditions in powerful explosions was performed in [3] for solution of the problem of shock-wave stability. In the present case, the medium behind the shock front is incompressible and there are no density perturbations. In analogy with [3], one can obtain the relations

$$\begin{cases} p = 2\rho_1 u_0 \dot{\varepsilon} - \frac{\partial p_0}{\partial x} \varepsilon; \\ u = \xi \dot{\varepsilon}; \\ v = u_0 \frac{\partial \varepsilon}{\partial y}, \end{cases}$$

when x = R, the term$(\partial p_0/\partial x)\varepsilon$ appears because of the drift of the boundary condition from the perturbed boundary at $x = X_b$ toward the old boundary at x = R.

On the free surface, we have two relations: from the condition for continuity of pressure

$$p = -\frac{\partial p_0}{\partial x} \delta$$

and from kinematical conditions

$$u = \dot{\delta} \text{ when } x = a.$$

We assume that the perturbations of the free boundary and of the surface of the shock wave are harmonic

$$\varepsilon = \varepsilon(t) \cos ky; \; \delta = \delta(t) \cos ky.$$

Then the boundary conditions are written in the form

at $x = a$

$$\begin{cases} p = \rho_2 \dot{u}_0 \delta \cos ky, \\ u = \dot{\delta} \cos ky, \end{cases} \quad (2)$$

at $x = R$

$$\begin{cases} p = (\rho_2 \dot{u}_0 \varepsilon + 2\rho_1 u_0 \dot{\varepsilon}) \cos ky, \\ u = \zeta \dot{\varepsilon} \cos ky, \\ du/dx = -u_0 k^2 \varepsilon \cos ky. \end{cases} \quad (3)$$

The last of the relations in (3) was obtained through the continuity equation.

Differentiating the first equation in (1) with respect to x and the second with respect to y, and adding them, we obtain a Laplace equation for p. Subsequently, the scheme for construction of a solution is the following: first, we seek a solution of the equation $\Delta p = 0$ satisfying the first conditions in (2) and (3); then, using this solution, we find the function u(x, y, t) from the equation of motion and having substituted it in the remaining boundary conditions, we obtain the desired relations for determination of the perturbation amplitudes $\varepsilon(t)$ and $\delta(t)$.

One can verify that the function

$$p = \frac{(\rho_2 \dot{u}_0 \varepsilon + 2\rho_1 u_0 \dot{\varepsilon}) \operatorname{sh} k(x-a) + \rho_2 \dot{u}_0 \delta \operatorname{sh} k(R-x)}{\operatorname{sh} k(R-a)} \cos ky$$

satisfies the necessary requirements for p.

The family of curves for the first equation of the system (1) have the form $\partial x/\partial t = u_0(t)$. Along the curves, $(\partial u/\partial t) = -(1/\rho_2)(\partial p/\partial x)$ and $u(x, t) = u(R, \tau) - \int_\tau^t \frac{1}{\rho_2} \frac{\partial p}{\partial x} [a(z) + x - a(t), z]\, dz$, where τ is the time of shock-wave arrival at the point with the coordinate $R(\tau)$ (Fig. 1).

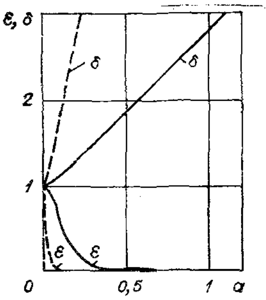

Fig. 2

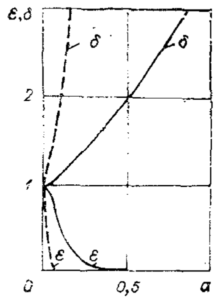

Fig. 3

$$R(\tau) - a(\tau) = x - a(t)$$

When x = a,

$$u(a, t) = u(R(0), 0) - \int_0^t \frac{1}{\rho_2} \frac{\partial p}{\partial x}(a(z), z)\,dz.$$

Differentiating this expression with respect to time and substituting it in the second relation in (2), we obtain

$$\ddot{\delta} \cos ky = -\frac{1}{\rho_2} \frac{\partial p}{\partial x}(a, t). \tag{4}$$

We determine $\partial u/\partial x$ when x = R,

$$\frac{\partial u}{\partial x} = \frac{\partial u(R, \tau)}{\partial \tau} \frac{\partial \tau}{\partial x} - \int_\tau^t \frac{1}{\rho_2} \frac{\partial^2 p}{\partial x^2}[a(z) + x - a(t), z]\,dz + \frac{1}{\rho_2} \frac{\partial p(R(\tau), \tau)}{\partial x} \frac{\partial \tau}{\partial x};$$

when x = R, t = τ,

$$\frac{\partial u}{\partial x} = \left[\frac{\partial u(R, t)}{\partial t} + \frac{1}{\rho_2} \frac{\partial p(R, t)}{\partial x}\right] \frac{\partial \tau}{\partial x};$$

$$\frac{\partial \tau}{\partial x} = \frac{1}{\dot{R} - \dot{a}} = \frac{\xi}{(1 - \xi) u_0}.$$

Using the second and third conditions in (3), we obtain

$$u_0 k^2 \varepsilon \cos ky = -\frac{\xi}{(1 - \xi) u_0}\left[\xi \ddot{\varepsilon} \cos ky + \frac{1}{\rho_2} \frac{\partial p(R, t)}{\partial x}\right]. \tag{5}$$

Substituting the expression for p(x, t) in Eqs. (4) and (5), we obtain a system of ordinary differential equations for the determination of δ and ε,

$$\begin{gathered} \frac{d^2\delta}{dt^2} + \frac{2k(1 - \xi) u_0}{\operatorname{sh} k(R - a)} \frac{d\varepsilon}{dt} + \frac{k u_0}{\operatorname{sh} k(R - a)} \varepsilon - k u_0 \operatorname{cth} k(R - a)\, \delta = 0; \\ \xi \frac{d^2\varepsilon}{dt^2} + 2k(1 - \xi) u_0 \operatorname{cth} k(R - a) \frac{d\varepsilon}{dt} + \left[\frac{u_0^2 k^2 (1 - \xi)}{\xi} + k u_0 \operatorname{cth} k(R - a)\right] \varepsilon - \frac{k u_0}{\operatorname{sh} k(R - a)} \delta = 0, \\ \varepsilon(0) = \delta(0). \end{gathered} \tag{6}$$

This system was solved numerically for three different types of motion:

uniform – u_0 = const, $a = u_0 t$;

uniformly accelerated – $u_0 = \alpha t$, $a = \alpha t^2/2$;

decelerated – $u_0 = \beta/\sqrt{2\beta t + 1}$, $a = \sqrt{2\beta t + 1}$.

The last solution for u_0 was obtained in the model of [1] for $p_s = 0$ and an adiabatically varying pressure at the free boundary:

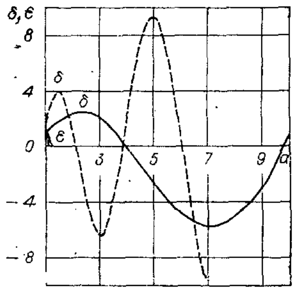

Fig. 4

$$p = p_{00}\left(\frac{a_0}{a}\right)^{\gamma} \quad \text{for } \gamma=3.$$

We investigate the behavior of the solutions of the system (6) for $t \to \infty$. One can then neglect in the equations terms containing the coefficient 1/sh k(R− a). The system of equations (6) takes the form

$$\frac{d^2\delta}{dt^2} - k\dot{u}_0\delta = 0;$$
$$\xi\frac{d^2\varepsilon}{dt^2} + 2k(1-\xi)u_0\frac{d\varepsilon}{dt} + \left[\frac{u_0^2k^2(1-\xi)}{\xi} + k\dot{u}_0\right]\varepsilon = 0. \quad (7)$$

In the case of uniform motion, Eqs. (7) have the simple solutions

$$\delta = A_1t + A_2;$$
$$\varepsilon = B_1e^{n_1t} + B_2e^{n_2t};$$
$$n_{1,2} = \frac{k(1-\xi)u_0}{\xi}\left(-1 \pm i\sqrt{\frac{\xi}{1-\xi}}\right); \; A_i, B_i - \text{const},$$

i.e., the perturbations of the free boundary increase linearly for large t and those in the shock wave are damped (damped oscillations). The results of a numerical calculation exemplifying this case are shown in Fig. 2, where the solid line corresponds to the value k = 2 and the dashed line, to k = 10.

For uniformly accelerated motion, the system (7) takes the form

$$\begin{cases}\ddot{\delta} - k\alpha\delta = 0;\\ \ddot{\varepsilon} + \dfrac{2k(1-\xi)\alpha t\dot{\varepsilon}}{\xi} + \alpha^2t^2k^2\dfrac{1-\xi}{\xi^2}\varepsilon = 0.\end{cases}$$

Then $\delta = C_1e^{\sqrt{k\alpha t}} + C_2e^{-\sqrt{k\alpha t}}$. In the second equation we make the substitution

$$\varepsilon = w(t)\exp\left[-\frac{k(1-\xi)\alpha}{2\xi}t^2\right].$$

We then obtain the equation

$$\ddot{w} + \left[\frac{k^2\alpha^2(1-\xi)}{\xi}t^2 - \frac{k(1-\xi)}{2\xi}\alpha\right]w = 0$$

for w(t). Neglecting the second term in the coefficient of w, we obtain

$$w = \sqrt{t}\,I_{\pm\frac{1}{4}}\left(k\alpha\sqrt{\frac{1-\xi}{\xi}}\frac{t^2}{2}\right).$$

On the basis of the material presented above, one can arrive at the following conclusion: for uniformly accelerated motion the perturbations at the shock front decrease exponentially and those at the free surface increase exponentially. Results of numerical calculations for k = 2 and k = 10 in this case are shown in Fig. 3.

For decelerated motion

$$a = \sqrt{2\beta t + 1}; \qquad u_0 = \frac{\beta}{\sqrt{2\beta t + 1}},$$

and the system (7) is written in the form

$$\begin{cases} \ddot{\delta} + \dfrac{k\beta^2}{(2\beta t + 1)^{3/2}}\delta = 0; \\ \ddot{\varepsilon} + \dfrac{2k(1-\xi)}{\xi}\dfrac{\beta}{\sqrt{2\beta t + 1}}\dot{\varepsilon} + \left[\dfrac{k^2\beta^2(1-\xi)}{(2\beta t + 1)\xi^2} - \right. \\ \left. - \dfrac{k\beta^2}{\xi(2\beta t + 1)^{3/2}}\right]\varepsilon = 0. \end{cases}$$

Neglecting the comparison with $2\beta t$ and neglecting the second term in the coefficient of ε, we obtain after the substitution $r = \sqrt{t}$

$$\begin{cases} r\dfrac{d^2\delta}{dr^2} - \dfrac{d\delta}{dr} + k\sqrt{2\beta}\delta = 0; \\ r\dfrac{d^2\varepsilon}{dr^2} + \left[\dfrac{2\sqrt{2\beta}k(1-\xi)}{\xi}r - 1\right]\dfrac{d\varepsilon}{dr} + \\ + \dfrac{2\beta k^2(1-\xi)}{\xi^2}r\varepsilon = 0. \end{cases}$$

Here also one can neglect the -1 term in the second part of the equation for ε. The system then has the following solution:

$$\varepsilon = D_1 e^{\gamma_1\sqrt{t}} + D_2 e^{\gamma_2\sqrt{t}};$$
$$\gamma_{1,2} = \frac{k(1-\xi)\sqrt{2\beta}}{\xi}\left(-1 \pm i\sqrt{\frac{\xi}{1-\xi}}\right);$$
$$\delta = \sqrt{t}\,J_{\pm 2}\left(2\sqrt{k\sqrt{2\beta t}}\right).$$

The solution for ε represents damped harmonic oscillations and that for δ represents harmonic oscillations whose amplitude increases as $\delta \sim t^{3/8}$ for large t. Curves for ε and δ are shown in Fig. 4 for k = 2 and k = 10 which illustrate this type of motion for short times.

In all the types of motion discussed, the shock wave is stable with respect to small perturbations and the free boundary is unstable with the growth rate of the instability depending on the form of the unperturbed motion. The instability grows most rapidly for accelerated motion and slowest of all for decelerated motion of the boundary of the explosion-produced cavity.

When producing explosions in the ground, it was long ago noted that a considerable volume of the ground around an explosion-produced cavity was penetrated by numerous radial cracks which emerged at the surface of the cavity. The observed instability is a possible "trigger mechanism" for crack formation during an explosion in the ground.

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